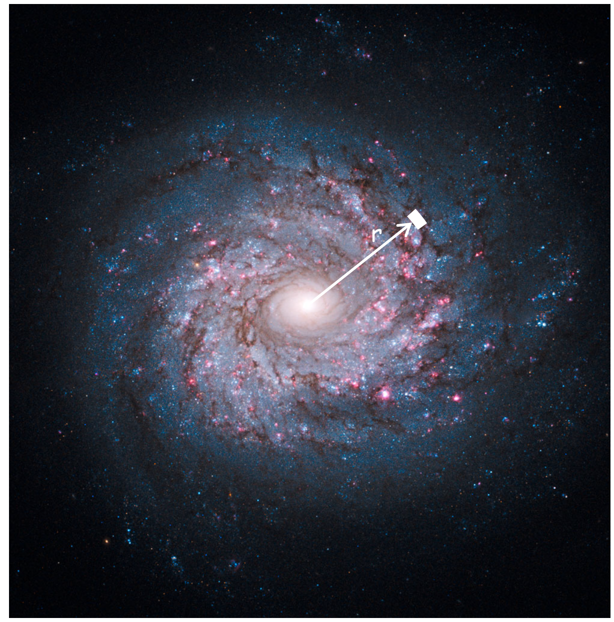
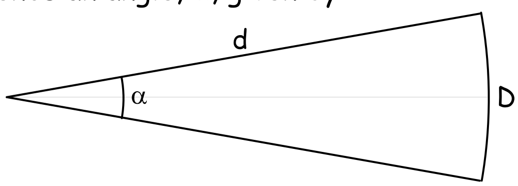
Galaxy Photometry



For galaxies, we measure a surface flux, that is, the counts in each pixel. Through calibration, this is converted to flux density in Janskys (1 Jy = 10^{-26} W/m^2/Hz).

For a galaxy[1] at some distance, d, a pixel of side D subtends an angle, α, given by

$$\alpha = \frac{D}{d}.$$



The surface brightness is the amount of light in that patch of sky divided by its area (in arsec^2):

$$I(r) = \frac{\text{Flux}}{\alpha^2} \sim \frac{\text{Jy}}{\text{arcsec}^2}.$$

Recalling the relationship between flux and luminosity, $\text{Flux} = \frac{\text{Luminosity}}{4\pi d^2}$, the surface brightness becomes

$$I(r) = \frac{F}{\alpha^2} = \frac{L}{4\pi d^2}\left(\frac{d}{D}\right)^2 = \frac{L}{4\pi D^2} \sim \frac{L_\odot}{\text{pc}^2} \text{ or } \frac{\text{Watts}}{\text{arcsec}^2}.$$

Which is often given in solar luminosities per parsec^2.

To convert this to magnitudes, recall that the apparent magnitude is a measure of flux,

$$m - m_0 = -2.5 \log\left(\frac{F}{F_0}\right).$$

So the surface brightness in magnitudes per arsec^2 is

$$\mu - \mu_0 = -2.5 \log\left(\frac{F/\alpha^2}{F_0/\alpha^2}\right),$$

$$\mu - \mu_0 = -2.5 \log\left(\frac{I}{I_0}\right).$$

EXAMPLE: For the Sun in the optical V-Band, taking $I_0 = 1\ L_\odot/\text{pc}^2$ for $\mu_0 = 26.4$ magnitudes/arcsec^2, yields

$$\mu = -2.5 \log(I) + 26.4 \sim \text{mag/arcsec}^2.$$

Giving the surface brightness (in the V-band in this case), as

$$I_V = 10^{0.4(26.4-\mu_V)} \frac{L_{\odot V}}{\text{pc}^2}.$$

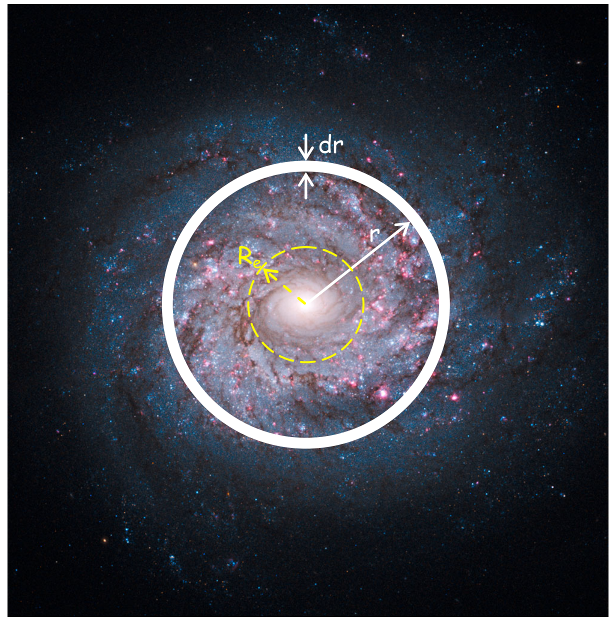
---

[1] From Amina Helmi's Kapteyn course at www.astro.rug.nl/~ahelmi/teaching/gal2010/ellipt.pdf

The total (apparent) magnitude of the galaxy is the surface brightness integrated over the entire galaxy.

To do this, we really need to model the galaxy since the surface brightness is uneven and the edge is very hard to determine. So astronomers use surface profiles, particularly those based on Sérsic[2] with developments by de Vaucouleurs and Petrosian and the exponential case.

The Sérsic Profile



Sérsic Profile (Graham & Driver, Whittle[3]) describes the intensity of a differential annulus at a distance R from the center of a galaxy:

$$I(r) = I_e e^{-b_n\left[\left(\frac{r}{R_e}\right)^{1/n} - 1\right]}$$

where I_e is the intensity in flux per arcsecond^2 ($f/\square^2$) at the effective radius R_e enclosing half the total light. The luminosity within some radius, R, is then the integral of this times the area of the differential annulus ($2\pi r\, dr$) from zero to R:

$$L(R) = \int_0^R 2\pi r I_e e^{-b_n\left[\left(\frac{r}{R_e}\right)^{1/n} - 1\right]} dr \qquad (1)$$

This results in an expression that includes an incomplete Gamma function,

$$L(<R) = 2n\pi I_e R_e^2 \frac{e^{b_n}}{b_n^{2n}} \gamma(2n, x) \quad \text{where} \quad \gamma(2n, x) = \int_0^x e^{-t} t^{2n-1} dt \qquad (2)$$

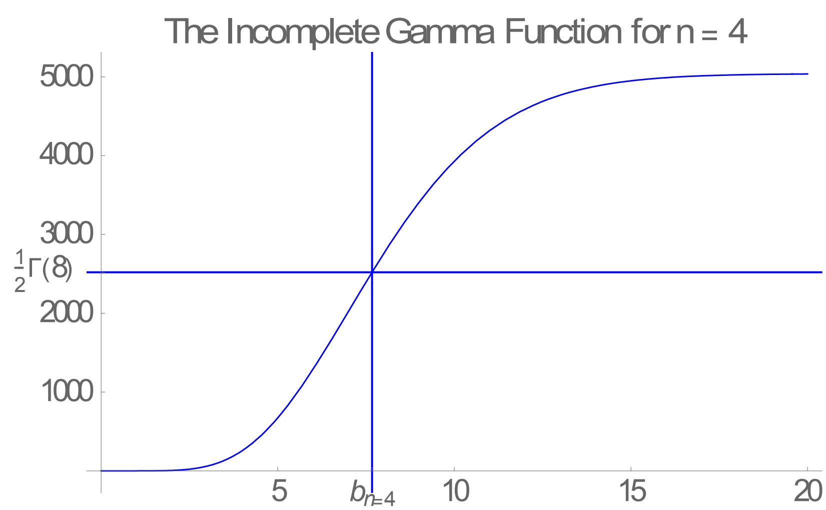
For n > 0, this has a solution of

$$\int_0^{b_n} e^{-t} t^{2n-1} dt = \gamma(2n, b_n) = \Gamma(2n) - \gamma(2n, b_n)$$

or

$$2\gamma(2n, b_n) = \Gamma(2n) \qquad (3)$$

The plot of $\gamma(2n, b_n)$ for n = 4 shows that $\frac{1}{2}\Gamma(2*4) = 2520$ corresponds to $b_n = 7.67$.



---

[2] Sérsic, J., L., 1968, *Atlas de Galaxieas Australes*. Observatorio Astronomico, Cordoba

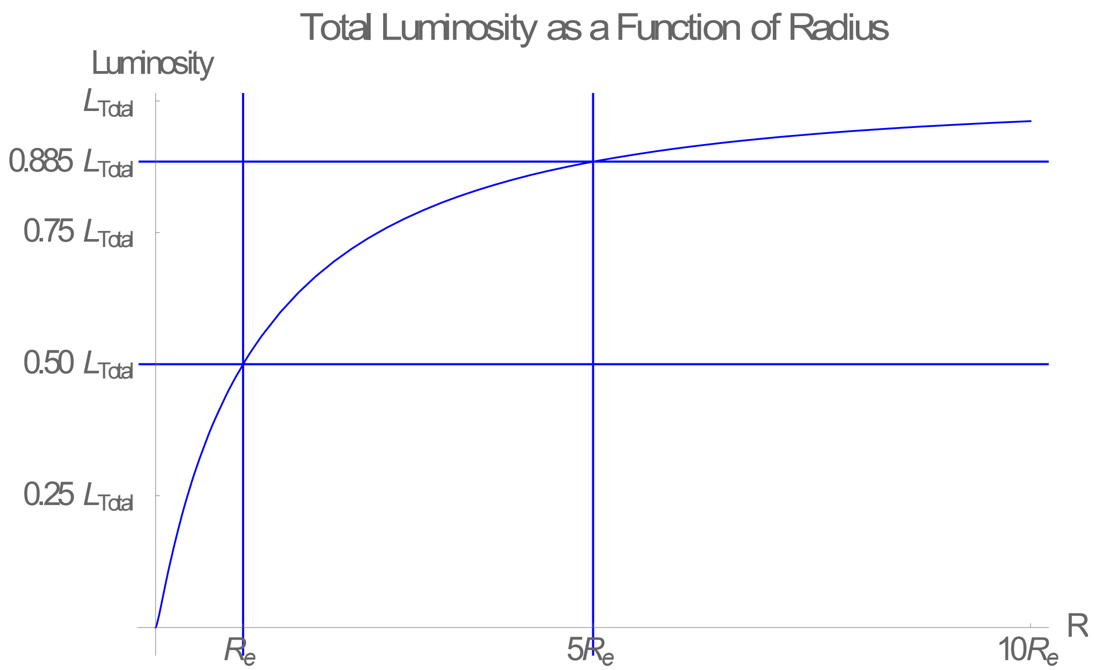
[3] adsabs.harvard.edu/abs/2005PASA...22..118G and

The de Vaucouleurs Profile

Replacing n = 4 and b_n = 7.67 in Equation (1) gives the de Vaucouleurs

$$I_{deV}(r) = I_e e^{-7.67\left[\left(\frac{r}{R_e}\right)^{1/4} - 1\right]} \quad (4)$$

Plotting the total luminosity as a function of radius, in arbitrary units, shows that half the total luminosity is at R_e and 88.5% of the total luminosity is within 5Re, as Graham & Driver (2005) point out.



The total luminosity of the galaxy is found by integrating to infinity:

$$L_{Tot} = \int_0^\infty 2\pi r I_e e^{-b_n\left[\left(\frac{r}{R_e}\right)^{1/n} - 1\right]} dr$$

Which results in a normal Gamma Function

$$L_{Tot} = 2n\pi I_e R_e^2 \frac{e^{b_n}}{b_n^{2n}} \Gamma(2n) = \pi I_e R_e^2 \frac{e^{b_n}}{b_n^{2n}} \left[2n\Gamma(2n)\right]$$

Since $\Gamma(2n) = (2n-1)!$, $2n*\Gamma(2n) = (2n)!$. Thus, for n = 4 and b_n = 7.669,

$$L_{Tot} = \pi I_e R_e^2 \frac{e^{7.669}}{7.669^8} (2*4)! = 7.215\ \pi I_e R_e^2. \quad (5)$$

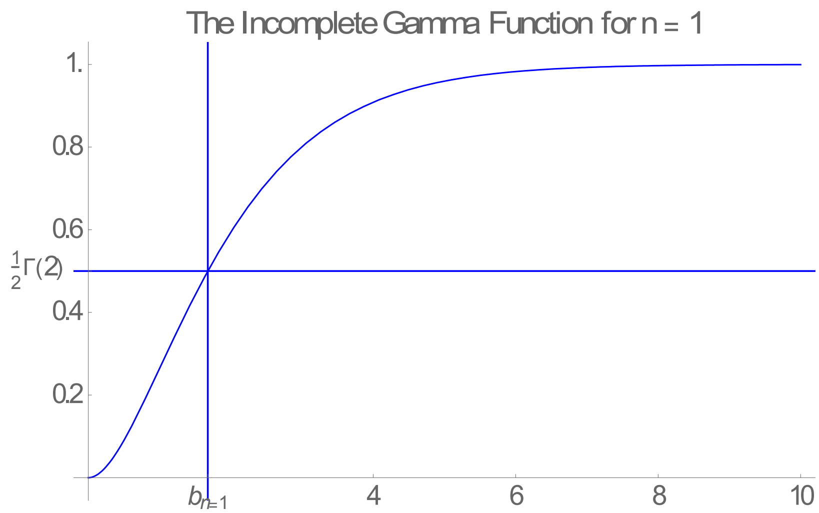
The Exponential Profile

If n = 1 is replaced in Equation (3)

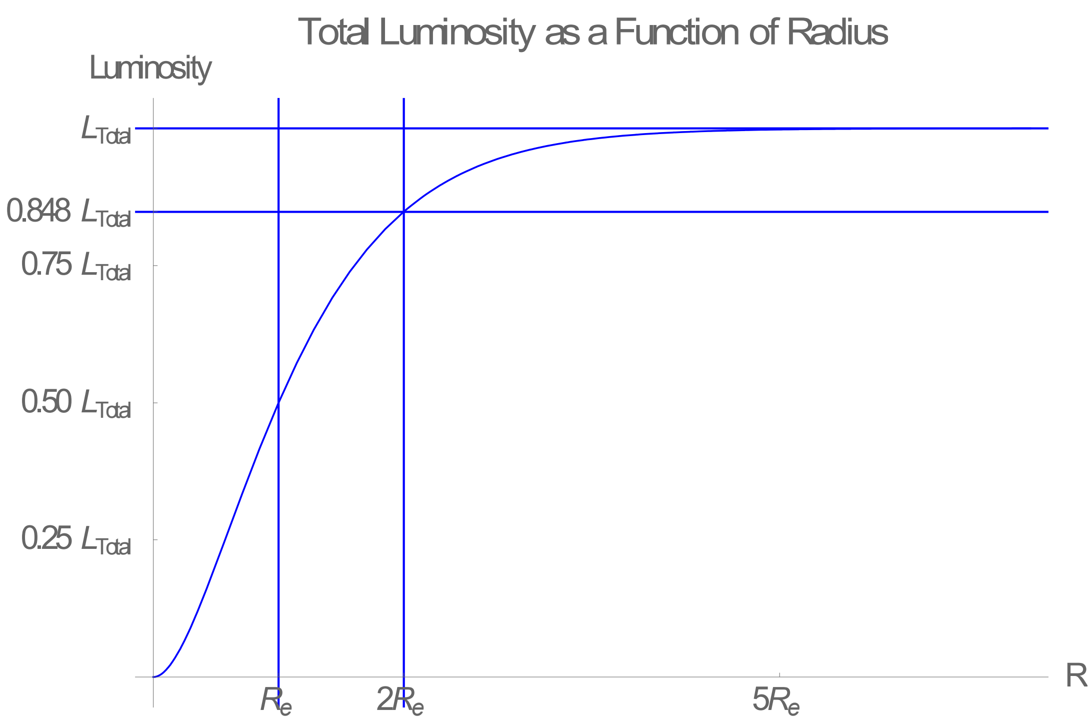
$$2\gamma\left(2n,b_n\right)=\Gamma\left(2n\right)$$

the plot of $\gamma(2n, b_n)$ for n = 1 shows that $\frac{1}{2}\Gamma(2*1) = 0.5$ corresponds to $b_n = 1.68$. Equation (1) then gives the exponential profile:

$$I_{exp}(r)=I_e e^{-1.68\left[\left(\frac{r}{R_e}\right)-1\right]} \qquad (6)$$



Plotting the total luminosity as a function of radius, in arbitrary units, shows that half the total luminosity is at R_e and 84.8% of the total luminosity is within 2Re.

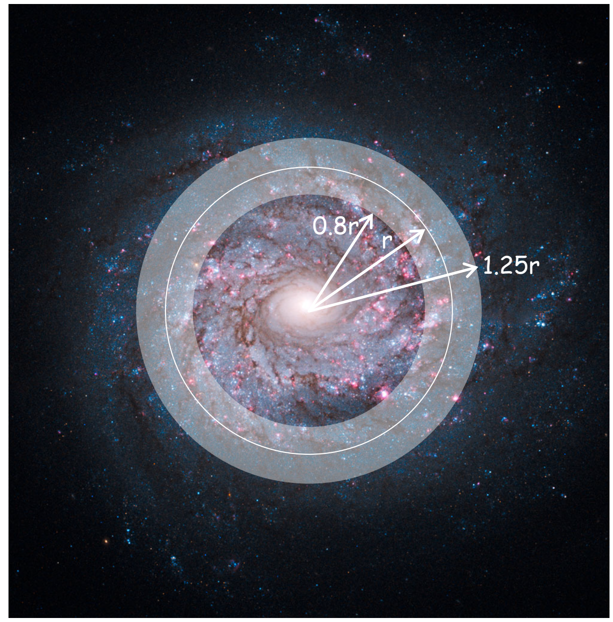


Galactic Scale Radius

For all of these models, the radius (eg., expRadr in SDSS) is the radius at which $I = I_0/e$.

The Petrosian Profile

The Petrosian Ratio used by SDSS is defined at a radius r from the center of an object. It is calculated from the integral of the ratio of the local surface brightness average over an annulus at r (SDSS takes it from 0.8r to 1.25r) to the total surface brightness within r. The "averaging" is just the total surface brightness within the annulus divided by the area of the annulus).

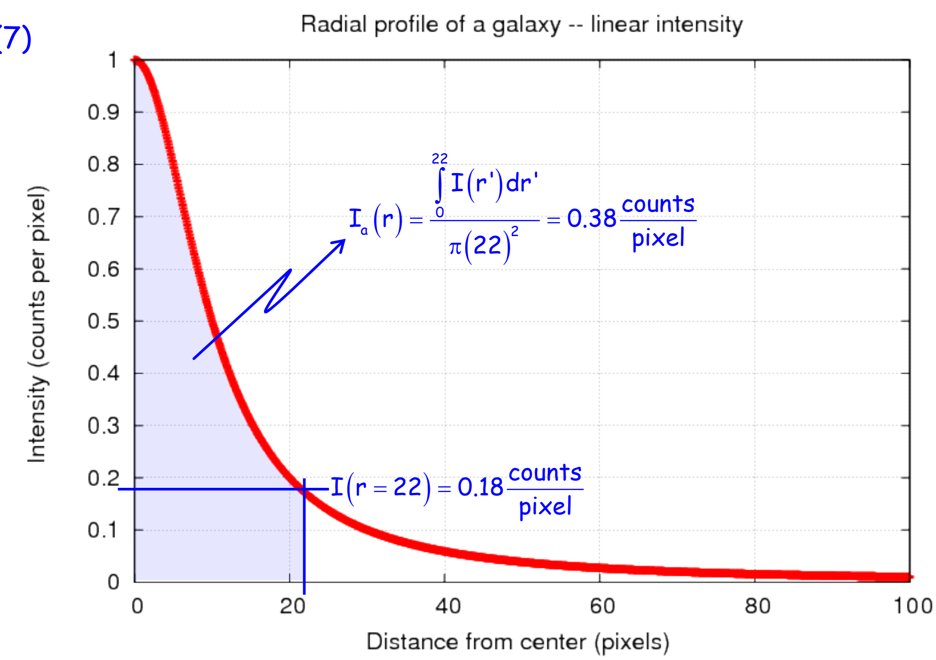


$$\mathscr{R}(r) = \frac{\left(\dfrac{\text{Total surface brighness within annulus}}{\text{Area within Annulus (0.8r - 1.25r)}}\right)}{\left(\dfrac{\text{Total surface brighness within r}}{\text{Area within r}}\right)}$$

Thus,

$$\mathscr{R}(r) = \frac{\dfrac{\int_{0.8r}^{1.25r} 2\pi r' I(r')dr'}{\pi\left(1.25^2 - 0.8^2\right)r^2}}{\dfrac{\int_0^r 2\pi r' I(r')dr'}{\pi r^2}} \qquad (7)$$

Plot from Michael Richmond's SPSP 240 course (http://stupendous.rit.edu/richmond/)



The radius at which this fraction is one is quite small, though it is formally defined as the Petrosian Radius. For the plot to the right, the surface brightness at 22 pixels is 0.18 counts/pixel (in the annulus 1 pixel wide) whereas the integrated intensity is about 0.38 counts/pixel. Going out to larger radii would decrease both values, but at some point, they would be equal and that would define the Petrosian Radius.

In practice, however, the Petrosian Radius turns out to be quite small, so SDSS (and others) define it as when the integrated flux is some fraction of the annular flux, η, that SDSS takes as $\eta = 0.2$

$$I(R_p) = \eta\mathscr{R} \qquad (8)$$

The total Petrosian flux is defined as the sum of all the flux within k times the Petrosian radius. The factor k is used to improve the signal-to-light ratio by including more of the galaxy's light than falls within R_P. SDSS takes k = 2 then defines the other Petrosian Radii as:

$R_{P,50}$ = radius containing half the total Petrosian Flux

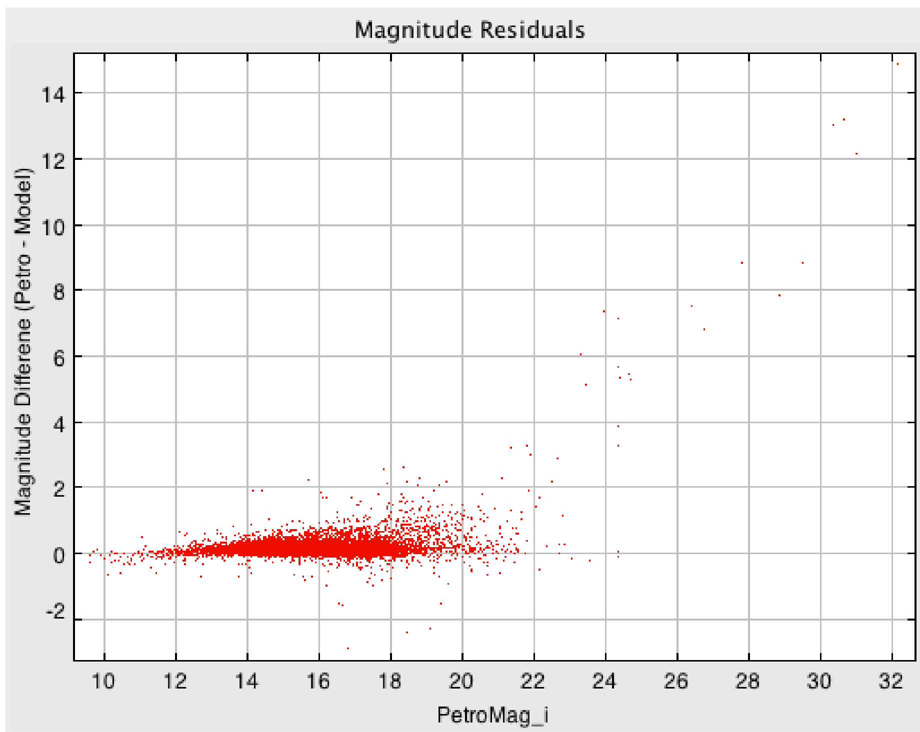
$R_{P,90}$ = radius containing 90% of the total Petrosian Flux

and the Petrosian concentration index is the ratio of these two radii:

$$C_P = \frac{R_{P,\,90}}{R_{P,\,50}} \qquad (9)$$

In the 2014 UAT Workshop Scavenger Hunt 3A, Martha states:

- While you might think that it would be easy to measure all the light, and hence the magnitude, it turns out that it's easier said than done, because the outer regions of the galaxy are fainter than the brightness of the night sky. So astronomers measure magnitudes in different ways. There are whole papers written on the subject of how to calculate magnitudes! **We rely on the recommendations of the SDSS staff to use "model magnitudes" when we want to calculate colors and "Petrosian magnitudes" when we want to calculate luminosities or absolute magnitudes.** Investigate the difference between the two for the observed i band magnitudes by plotting the residual PetroMag_I - ModelMag_I versus the Petrosian magnitude; examining residuals is a good way to look for trends. *Hint: Use the arithmetic capability in TOPCAT to generate a new columns as needed.*



The plot shows the Magnitude residuals are not zero, which they would be if there were no difference. Since the difference is more positive than negative, the Petrosian Magnitudes are larger than the Model Magnitudes, particularly for dimmer galaxies. This means that the galaxies are brighter in the model magnitudes than the Petrosian Magnitudes.